How to Solve Math Problems
A Common Sense Approach

Denise Gaskins
Author of Let's Play Math
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When a kid is feeling bad about being stuck with a problem, or just very anxious, I sometimes ask him to make as many mistakes as he can, and as outrageous as he can.

Laughter happens (which is valuable by itself, and not only for the mood—deep breathing brings oxygen to the brain).

Then the kid starts making mistakes.

In the process, features of the problem become much clearer, and in many cases a way to a solution presents itself.

—MAria DrOujkOva
Archimedes tried to find the value of π and almost discovered calculus. Pierre de Fermat calculated the result of a gambling game and laid the foundations of probability. Leonhard Euler went for an afternoon walk over the bridges of Königsberg and invented topology. Georg Cantor created a way to count infinity and opened up a whole new world of modern math. Through the centuries, mathematics has grown as mathematicians struggled with and solved challenging problems.

If we want our children to learn math the mathematician’s way, we need to offer them plenty of problems to solve. A child may work through several pages of number calculations by rote, following memorized steps, but a good problem demands more attention. A story problem puts flesh on the abstract bones of arithmetic, encouraging the child to ponder what it means for one thing to be bigger than another, or smaller, or faster, or slower, or made up of several parts.

According to American math professor Howard Eves, “There is a distinction between what may be called a problem and what may be considered an exercise. The latter serves to drill a student in some technique or procedure and requires little, if any, original thought. An exercise, then, can always be done with reasonable dispatch and with a minimum of creative thinking.”

Eves continued: “In contrast to an exercise, a problem, if it is a good one for its level, should require thought on the part of the student.”

Math professor Herb Gross says: “As important as mathematics is, it is a distant second to the need for good reading comprehension. We teachers so often hear students summarize a course by saying, ‘I could do everything
except the word problems.’ Sadly, in the textbook of life, there are only word problems.”

The more we can work with our children on reading, paraphrasing, and translating word problems into mathematical expressions, the better prepared they will be to face the challenges they will meet in future math classes and in the textbook of life.

Here is a four-step approach that will help children at any grade level think their way through a tough problem. The following questions represent a common-sense approach to solving any problem, which means students can learn to ask them for themselves.

In my co-op math classes, I print out a copy for each child to keep handy. I made individual “whiteboards”—a clear page protector filled with white card stock or several sheets of typing paper. Dry-erase markers wipe right off, and we can insert graph paper or other templates for the lesson of the day. Then I printed the problem-solving questions and slipped that into the flip side of each packet.

**Ask Yourself These Four Questions**

♦ What do I know?
♦ What do I want?
♦ What can I do?
♦ Does it make sense?

Let’s look closer at each question …
To Solve a Math Problem, Ask Four Questions

(1) What Do I Know?

Read the problem carefully. Reread it until you can describe the situation in your own words.

List the facts or information given in the problem.
Underline or circle any important words, such as factor, multiple, area, or perimeter. What do you remember about those topics?
Watch out for mixed units!
Express the facts in math symbols, if you can.

(2) What Do I Want?

Describe the goal, what the problem is asking you to find.
Underline or circle any important words, such as sum, product, next, or not. Small words like “not” are especially easy to miss.
Express the goal in math symbols, if you can.

(3) What Can I Do?

Combine the given facts. Even if you can’t solve the problem, can you think of a way to get closer to your goal?
Take one little step at a time.
Try a tool from your Problem Solving Toolbox:
♦ Draw a diagram or picture.
♦ Act the problem out, step by step.
Make a systematic list, chart, or table.
Look for a pattern.
Simplify the problem. (Try it with smaller numbers.)
Restate the problem in another way, or look for a related problem.
Think about “Before” and “After” situations.
Work backwards.
Guess and check. (Try something to see if it works.)

(4) Does It Make Sense?

Don’t neglect this last step! When you think you have found the answer, always look back at the original problem one more time.
Do you have the correct units (inches, cm², kg, etc.)?
Does your answer make sense?
Can you think of a way to confirm that your answer is right?

You might like to print out the following reference page for your students.
To Solve a Math Problem, Ask Four Questions

(1) What Do I Know?
Read the problem carefully. Reread it until you can describe the situation in your own words.
List the facts or information given in the problem.
Underline or circle any important words, such as factor, multiple, area, or perimeter. What do you remember about those topics? Watch out for mixed units!
Express the facts in math symbols, if you can.

(2) What Do I Want?
Describe the goal, what the problem is asking you to find.
Underline or circle any important words, such as sum, product, next, or not. Small words like “not” are especially easy to miss.
Express the goal in math symbols, if you can.

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Combine the given facts. Even if you can’t solve the problem, can you think of a way to get closer to your goal?
Take one little step at a time. Try a tool from your Problem Solving Toolbox:
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Does your answer make sense?
Can you think of a way to confirm that your answer is right?

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The teacher who wishes to serve equally all his students, future users and nonusers of mathematics, should teach problem solving so that it is about one-third mathematics and two-thirds common sense.

—George Polya
I love story problems. Like a detective, I enjoy sifting out clues and solving the mystery. But what do you do when you come across a real stumper? Acting out story problems could make a one-page assignment take all week.

You don't have to bake a pie to study fractions or jump off a cliff to learn gravity. Use your imagination instead. The following suggestions will help you find the clues you need to solve the case.

Step One: Understand the Problem

The first step is to visualize the story behind the problem. That is hard, because it is not something you can touch; it is an idea. But ideas are real and important parts of God’s world.

Read the problem. Read it again. Close your eyes and see the story in your mind. Now open your eyes and make a list of everything in the problem. What do you know? What do you need to find?
Draw a picture and label it. A picture will often help you think through the problem. What does $x$ stand for, and where is it on your picture? What is the angle measurement?

If you have several pieces of related data, try making a chart or graph. These can help you spot patterns that may lead to a solution.

**Step Two: Apply Common Sense**

Imagine yourself in the story situation. If it actually happened to you, what would you do?

Next, start to mix things around in your mind. Have you worked a problem like this before? How did you solve that one? Will that method, or something like it, work here?

What are you trying to find? Have you done any other problems with this type of unknown? How did you find the answer then?

Is there a formula that might help? Maybe the area or volume of the diagram? What do you get if you subtract these two numbers, or multiply them? Will that get you any closer to your answer? How do you know?

Look for an equation that has a variable for the answer you want (say, $t$ for time) and nothing else except things you know. Sometimes you cannot get it in one step. You may need to figure out what $d$ equals before you can calculate $t$. 
If you are completely stumped, try explaining the problem to someone. Talking it over might help unclog your brain.

Be careful not to scramble units. If one length is given in inches and another is given in yards, make them both inches or both yards.

You may not need all the numbers or facts given in the problem. Sometimes textbooks include extra information to give practice in deciding which facts you need for what you are trying to solve.

Don’t give up. Expect to make a few false starts. If you can solve a problem fast, it is not much of a challenge. But if you can solve a hard problem, one that really makes you think, then you know you are a master math detective.

Step Three: Present Your Case

When you think you have the answer, ask, “Does it make sense?” Can you find a way to check it?

Can you think of another way you might have gotten the answer? If you see an alternate approach, would that method have been easier? Make a note of any ideas you come up with. You may need them on your next case.

Finally, getting the right answer is of little use if the Chief Inspector can’t tell what you did. Make it a habit to communicate clearly and neatly. Write out each step of your work, and label your answer carefully, especially the units.

Case Closed

The detective goes on to his next challenge…
I’ve spent the last two days going over my problem, going over my approach, finding new gaps in my proof, fixing them … wash, rinse, repeat.

It’s amazing that this vision of math as “getting to the right answer on your first try” even exists. I have to make, unmake, remake so many mistakes to get where I’m going.

I think all mathematicians work that way.

Math doesn’t happen in a straight line. If I hadn’t made as many mistakes in my thinking about this problem, I don’t think I would have solved it.

Somehow, a big part of the experience of math is trouble. Frustration is the status quo.

But when you get something — the thrill!

— Dan Finkel
Answer-Getting vs. Problem Solving

Answer-getting asks “What is the answer?”, decides whether it is right, and then goes on to the next question. Relational problem solving asks “Why do you say that?” and listens for the student to explain his thinking. Problem solving is less interested in “right” or “wrong”—it cares more about “makes sense” or “needs justification.”

Sometimes my children and I don’t even bother to work out the calculation at the end of a math problem, because the thing that intrigues us is the web of interrelated ideas we find along the way:

♦ How can we recognize this type of problem?
♦ What other problems are related to it, and how they help us understand this one? Or can this problem help us figure out those others?
♦ What could we do if we had never seen a problem like this one before? How would we reason it out?
♦ Why does the formula work? Where did it come from, and how is it related to basic principles?
♦ What is the easiest or most efficient way to manipulate the numbers? Does this help us see more of the patterns and connections within our number system?
♦ Is there another way to approach the problem? How many different ways can we think of? Which way do we like best, and why?
Story Problem Challenge

I love to encourage my math club students to make up their own word problems. They don't have to know how to answer their problems, because we pass the stories around and work on them together.

Your family may enjoy trying some of the problems my students have made up. For privacy's sake, I have given each student a nickname; the age listed is when they wrote or dictated their stories.

From Chickenfoot, Age 6

I spent $8, and I had $4 left. How much money did I start with?

From Princess Kitten, Age 7½

Kelly’s family had 5 real dogs. Two of them got lost. Kelly went looking for them. She found 2 stuffed animals that looked like the 2 dogs that got lost. She went back to her home.

Then their 3 dogs went looking for the 2 dogs that got lost, and they came back with the 2 dogs and 3 stuffed animals that looked just like the 3 dogs. How many dogs are there altogether?

From One of the Three Musketeers, Age 11

Mrs. Sterns has two different recipes of cookies she is going to make. She sees that she does not have any chocolate chips left, so she has to go to the store. But first she has to find out how much to get. One recipe calls for 63 oz. The other calls for 52 oz.
When she gets to the store, the only packages they have are 20 oz. of chocolate chips. How many packages should she buy?

**From the Cowgirl, Age 12**

I am an odd number less than 50. I’m square, not prime. And I am divisible by 3. What number am I?

**From a Musketeer’s Sister, Age 13**

I take eight counts to do a pirouette turn and two fouettés turns. A pirouette takes 1/2 of the eight counts. One fouetté takes half that time. How many counts would it take to do the famous combination of one pirouette and 32 fouettés?

**From Computer Geek, Age 14**

You make about $27 profit from selling pineapple soda. If each can costs you 25 cents, and you bought 75 cans and sold 46 cans, how much did you sell each can of soda for?

**From the Engineer, Homeschooling Father**

When we went on a fishing vacation to Canada, we found out the grocery stores sold milk in large, 4-liter plastic bags instead of in gallon jugs. Inside each large bag there were three smaller plastic bags. How much milk did each of these smaller bags hold?

**And My Contribution**

The day before the great battle at the Black Gate, a company of 450 orcs camped among the host of Mordor. But an argument broke out over dinner, and 1/3 of them were killed. Then 2/5 of the remainder died when a drunken troll stumbled through their camp during the night. How many of the orcs survived to join the morning’s battle?
**Word problems are very valuable** in teaching mathematics not only to master mathematics, but also for general development. Especially valuable are word problems solved with minimal scholarship, without algebra, even sometimes without arithmetics, just by plain common sense. The more naive and ingenuous is the solution, the more it provides the child’s contact with abstract reality and independence from authority, the more independent and creative thinker the child becomes.

…

When we teach children to solve problems in school, we do not expect them to meet exactly and literally the same problems in later life. Mathematical education would be next to useless if its only use were literal. We want much more, we want to teach children to solve problems in general. In this respect traditional word problems are especially valuable, because to solve a word problem, you have to understand what is said there. This function of word problems is very poorly understood in America.

…

The main educative value of word problems is that they serve as mental manipulatives, paving children’s road to abstract thinking. Pumps and other mechanical appliances are easy to imagine working at a constant rate. Problems involving rate and speed should be (and in Russia are) common already in middle school. Trains, cars and ships are so widely used in textbooks not because all students are expected to go into transportation business, but for another, much more sound reason: these objects are easy to imagine moving at constant speeds.

—Andrei Toom
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http://denisegaskins.com/

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https://www.flickr.com/photos/chrstphre/2166815376
Maria Droujkova quote from a Natural Math Forum discussion of math club activities, February 3, 2011.
https://groups.google.com/d/msg/naturalmath/yW5Pdr8_WhI/h9xULrj1-4wJ
Herb Gross. Quoted by Jerome Dancis, “Reading Instruction for Arithmetic Word Problems.”
http://www2.math.umd.edu/~jnd/subhome/Reading_Instruction.htm
Sketch of Sherlock Holmes by Sidney Paget (1860-1908) [Public domain] via Wikimedia Commons.
William Powell (left) & John Barrymore in Sherlock Holmes (cropped screenshot, 1922) [Public domain] via Wikimedia Commons.
http://mathforlove.com/2009/02/82223587
http://www.de.ufpe.br/~toom/travel/sweden05/WP-SWEDEN-NEW.pdf
Also check out Andrei Toom’s articles on math education and humanities:
http://www.de.ufpe.br/~toom/my_articles/engeduc/index.htm
Books by Denise Gaskins

Let’s Play Math:
How Families Can Learn Math Together—and Enjoy It

The Math You Can Play Series:
Counting & Number Bonds
Addition & Subtraction
Math You Can Play Combo
Multiplication & Fractions
Prealgebra & Geometry (upcoming)

Reviews
“In a culture where maths anxiety is now a diagnosable problem, this book shows the way to maths joy.”

“With this approach I can teach my kids to think like mathematicians without worrying about leaving gaps.”

“There were so many parts of this book that I highlighted that I really gave my Kindle a workout!”

“Most of the games can easily be scaled up for older kids, teens, and even adults. These are not drills disguised as games, but activities that require problem solving and strategy as well as calculation.”
Let’s Play Math:
How Families Can Learn Math Together
—and Enjoy It

All parents and teachers have one thing in common: we want our children to understand and be able to use math. Filled with stories and pictures, Let’s Play Math offers a wealth of practical, hands-on ideas for exploring math concepts from preschool to high school.

Your children will gain a strong foundation when you approach math as a family game, playing with ideas. Sections include:

How to Understand Math: Introduce your children to the thrill of conquering a challenge. Build deep understanding by thinking, playing, and asking questions like a mathematician.

Playful Problem Solving: Awaken your children’s minds to the beauty and wonder of mathematics. Discover the social side of math, and learn games for players of all ages.

Math with Living Books: See how mathematical ideas ebb and flow through the centuries with this brief tour through history. Can your kids solve math puzzles from China, India, or Ancient Egypt?

Let’s Get Practical: Fit math into your family’s daily life, help your children develop mental calculation skills, and find out what to try when your child struggles with schoolwork.

Resources and References: With these lists of library books and Internet sites, you’ll never run out of playful math to explore.

Denise Gaskins provides a treasure trove of helpful tips for all families, whether your children are homeschooling, unschooling, or attending a traditional classroom. Even if you struggled with math in school, you can help your kids practice mental math skills, master the basic facts, and ask the kind of questions that encourage deeper thought.

Don’t let your children suffer from the epidemic of math anxiety. Grab a copy of Let’s Play Math, and start enjoying math today.
Are you tired of the daily homework drama? Do your children sigh, fidget, whine, stare out the window—anything except work on their math? Wouldn’t it be wonderful if math were something your kids wanted to do?

With the Math You Can Play series, your children can practice their math skills by playing games with basic items you already have around the house, such as playing cards and dice.

Math games pump up mental muscle, reduce the fear of failure, and develop a positive attitude toward mathematics. Through playful interaction, games strengthen a child’s intuitive understanding of numbers and build problem-solving strategies. Mastering a math game can be hard work, but kids do it willingly because it’s fun.

So what are you waiting for? Clear off a table, grab a deck of cards, and let’s play some math!

**Counting & Number Bonds: Math Games for Early Learners**

Preschool to Second Grade: Young children can play with counting and number recognition, while older students explore place value, build number sense, and begin learning the basics of addition.

**Addition & Subtraction: Math Games for Elementary Students**

Kindergarten to Fourth Grade: Children develop mental flexibility by playing with numbers, from basic math facts to the hundreds and beyond. Logic games build strategic thinking skills, and dice games give students hands-on experience with probability.
Math You Can Play Combo: Number Games for Young Learners

Preschool to Fourth Grade: A combined volume, two books in one, with 42 kid-tested games that offer a variety of challenges for preschool and school-age learners. Help your children master the math facts and build a foundation for future learning.

Multiplication & Fractions: Math Games for Tough Topics

Second to Sixth Grade: Students learn several math models that provide a sturdy foundation for understanding multiplication and fractions. The games feature times table facts and more advanced concepts such as division, fractions, decimals, and multistep mental math.
Denise Gaskins enjoys math, and she delights in sharing that joy with young people. “Math is not just rules and rote memory,” she says. “Math is like ice cream, with more flavors than you can imagine. And if all you ever do is textbook math, that’s like eating broccoli-flavored ice cream.”

A veteran homeschooling mother of five, Denise has taught or tutored mathematics at every level from pre-K to undergraduate physics. “Which,” she explains, “at least in the recitation class I taught, was just one story problem after another. What fun!”

Now she writes the popular blog Let’s Play Math and manages the Math Teachers at Play monthly math education blog carnival.

*Want to help your kids with math? Don’t help with the homework. Get them to engage with math by doing things together—many of which don’t even look like math. Let’s Play Math is charming, intelligent, and practical; full of family fun and sound advice.*

—Ian Stewart

*author of Professor Stewart’s Casebook of Mathematical Mysteries*

*Reading one of Gaskins’ books is like going to a really great teacher workshop—part philosophy, part practical ideas, and all excellent. She just oozes expertise and enthusiasm.*

—Amy at hopeisthewordblog.com